Soldiers, forty centuries are looking down upon you.
—Napoleon Bonaparte in Egypt

Math Club by Mail

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Inside this issue of Math Club by Mail:

Alexandria Jones and Secret of the Egyptian Fractions

Archaeology professor Dr. Fibonacci Jones came home from a long day of lecturing and office work. Stepping inside the front door, he held up a shiny silver disk.

"Ta-da!" he said.

"All right!" said his daughter Alex. "The photos are here."

They had to chase Alex's brother Leon off the computer so they could view the CD-ROM, but that wasn't hard. He wanted to see the artifacts, too.

Alex recognized several of the items they had dug up from the Egyptian scribe's burial plot—the wooden palette, some clay pots, and of course the embalmed body. Then came several close-up pictures of writing on papyrus.

"I remember how to read the Egyptian numbers," Alex said, "but what are these marks above them?"

Dr. Jones nodded. "I thought you'd catch that. Those are fractions. The scribe places an open mouth, which is the hieroglyph "r," over a number to make its reciprocal."

"I know that word," Leon said. "It means *one over* the number. Like, the reciprocal of 12 is 1/12, right?"

"That's right. Twelve would be written as $||\cap$, and the fraction $\overset{\circ}{||\cap}$ means 1/12. When we write it in translation, we just put a line over the number: $\overline{12}$."

"I get it," Alex said. "So 1/2 would be written as $\overline{2}$, and 1/4 is $\overline{4}$. That's pretty easy. But what about a bigger fraction, like 3/4? You can't write that as 'one over' something."

"Good point," Dr. Jones said. "The scribes wrote many fractions as sums. We write a mixed number as the sum of a whole number and a fraction. They did the same with their fractions."

"So, 3/4 is $\overline{2} + \overline{4}$?"

"Yes, without the plus sign—just like we don't use "+" when we write a mixed number."

"Okay, 3/4 = $\overline{2}\ \overline{4}$," Alex said.

"That's right. Except..."

"I knew it!" Leon said. "It's never that easy."

Dr. Jones laughed. "I guess it isn't. You see, the Egyptians had a favorite number, which they used whenever they could. Even though 3/4 = $\overline{2}\ \overline{4}$ seems obvious to us, they'd probably use something else."

(Continued on page 2)

Archaeologists can translate Egyptian math because the Rosetta stone serves as a language key. It was inscribed in hieroglyphs, Egyptian cursive, and Greek.

Alexandria Jones, Continued

"We give up," Alex said. "What's the mystery number?"

"The scribes' favorite number was 2/3. It even had a special symbol:  We write it in translation as $\bar{\bar{3}}$, with two lines on top."

"Weird!" Leon said. "Why 2/3?"

"We don't know. Maybe they thought it was magic or something."

"And 3/4 is more than 2/3," Alex said, "so they could use that in their sum. Let's see. The common denominator is 12...here we go: 3/4 = $\bar{\bar{3}}\ \overline{12}$, right?"

"I've got one that doesn't use 2/3," Leon said. "How about 2/5 = $\bar{5}\ \bar{5}$. This isn't so hard."

"Be careful," said his father. "A scribe would think that answer was childish. They never used the same fraction twice in one number."

"No fair!" Leon said. "Well, I could convert the second fifth to tenths, but that's 2/5 = $\bar{5}\ \overline{10}\ \overline{10}$, which still repeats."

"And if you changed the tenth to twentieths," Alex said, "you still have trouble: 2/5 = $\bar{5}\ \overline{10}\ \overline{20}\ \overline{20}$. You could go on forever."

"Fractions with a two in the numerator were always hard for the scribes." Dr. Jones smiled. "But they were especially important because the Egyptians did so much of their math with the times-two tables."

"So how did they solve the problem?" Alex asked.

"The scribes made reference charts," Dr. Jones said. "The Rhind papyrus contains a long list of double fractions, from 2/3 to 2/101. For 2/5, they used—"

"No! Don't tell us," Leon said. "I think I know: 2/5 = 6/15 = 5/15 + 1/15. And 5/15 is the same as 1/3. That means 2/5 = $\bar{3}\ \overline{15}$."

"Very good. That's exactly the value the scribes used."

"Okay, Dad, I can do the next one," Alex said. "If 2/5 = $\bar{3}\ \overline{15}$, then 3/5 is just one more fifth: 3/5 = $\bar{3}\ \overline{15}\ \bar{5}$. Or should it be $\bar{3}\ \bar{5}\ \overline{15}$?"

"The latter," Dr. Jones said. "Always write the fractions from smallest number to largest. But there's a way to write 3/5 with only two terms. Can you figure it out?"

"Hmm. 3/5 is a little more than a half, isn't it?" Alex bit her lip and looked at the ceiling for a minute. "Of course. It's 6/10, which is (5+1)/10. So 3/5 = $\bar{2}\ \overline{10}$."

Dr. Jones grinned proudly at his children. "I do believe you've got it."

In 1798 Napoleon invaded Egypt, and he decided to raid her treasures in the name of science. He chose a group of scholars to help him explore and pick through the plunder. Their greatest find was the Rosetta Stone, torn from an old wall on the western bank of the Nile. In 1824, Jean-Francois Champollion succeeded in breaking the hieroglyphic code—but it took nearly 100 years to get a full translation.

Tell a friend about Math Club!

"Math is a game, playing with ideas."

MATH CLUB RULE: FIGURE IT OUT FOR YOURSELF.

You will notice there are no answers given to any of the puzzles in this issue. The answers will be in the next issue, but I hope you will figure them out a long time before that.

A cone looks like a triangle from the side and a circle from the top or bottom. What shape looks like a triangle from any side?

Gaspard Monge (1746-1818) started as a drafts-man who did geometry in his spare time. He took up chemistry and helped Lavoisier with experiments on water. Later, he served as president of Napoleon's Institut d'Egypte in Cairo. Monge applied his drafting skills to create the science of *descriptive geometry*—the study of projections (that is, how to draw 3-D curves and shapes in two dimensions).

Alex Plays With Fractions

Alex decided that making Egyptian-style fractions was a fun challenge. After she finished the supper dishes, she spread a large sheet of paper on the kitchen table and started to make a poster. First she wrote in all the fractions she could fit on the sheet, from 1/2 to 9/10. Then she filled in the chart, starting with the fractions she and Leon had found earlier.

Can you finish Alex's chart?

Egyptian fraction rules:

1. Use 2/3 whenever you can.
   3/4 = $\overline{\overline{3}}\ \overline{12}$, not $\overline{2}\ \overline{4}$
2. Never repeat terms.
   2/5 = $\overline{3}\ \overline{15}$, not $\overline{5}\ \overline{5}$
3. Write from smallest number to largest.
   3/5 = $\overline{3}\ \overline{5}\ \overline{15}$, not $\overline{3}\ \overline{15}\ \overline{5}$
4. Use the simplest combination you can.
   3/5 = $\overline{2}\ \overline{10}$, not $\overline{3}\ \overline{5}\ \overline{15}$

1/2 = $\overline{2}$

1/3 = $\overline{3}$ 2/3 = $\overline{\overline{3}}$

1/4 = $\overline{4}$ 2/4 = 3/4 = $\overline{\overline{3}}\ \overline{12}$

1/5 = 2/5 = $\overline{3}\ \overline{15}$ 3/5 = $\overline{2}\ \overline{10}$ 4/5 =

1/6 = 2/6 = 3/6 = 4/6 = 5/6 =

1/7 = 2/7 = 3/7 = 4/7 = 5/7 =
6/7 =

1/8 = 2/8 = 3/8 = 4/8 = 5/8 =
6/8 = 7/8 =

1/9 = 2/9 = 3/9 = 4/9 = 5/9 =
6/9 = 7/9 = 8/9 =

1/10 = 2/10 = 3/10 = 4/10 = 5/10 =
6/10 = 7/10 = 8/10 = 9/10 =

Scientists use Fourier series to describe many types of repeating phenomena—things like light, sound, and other wave-like forms of energy.

A Trick for Solving Fractions

When Alex worked on her fraction chart, she tried to picture the fractions in her head. A good mental picture is much easier to work with than abstract numbers.

Alex's favorite fraction trick is to imagine the fractions as minutes on a clock. Because the clock face comes with built-in divisions (the numbers 1-12), she can easily picture several common fractions. Most obvious are the half and quarter hour divisions (3, 6, and 9) to mark the fractions 1/4, 1/2, and 3/4.

Thirds are easy, too. One third of an hour is 20 minutes. Alex uses the numbers 4 and 8 on the clock to mark the fractions 1/3 and 2/3.

And since thirds cut in half make sixths (and 2 is half of 4), she knows that the number 2 marks 1/6 of the way around the clock.

Another way she pictures the sixths is this: Ten minutes is 1/6 of 60 minutes. 20 minutes is 2/6 of an hour, and 30 minutes is 3/6.

By thinking this way, Alex can see some connections between different types of fractions. For instance, 30 minutes is half an hour, but it is also 20 minutes plus ten more minutes. That means that 1/2 = 1/3 + 1/6, or in Egyptian-style fractions, $\overline{2} = \overline{3}\ \overline{6}$.

Or what about the Egyptian scribes' favorite fraction, 2/3? The number 8 marks 2/3 of the way around the clock face, which is half an hour plus ten minutes more. This means that $\overline{\overline{3}} = \overline{2}\ \overline{6}$.

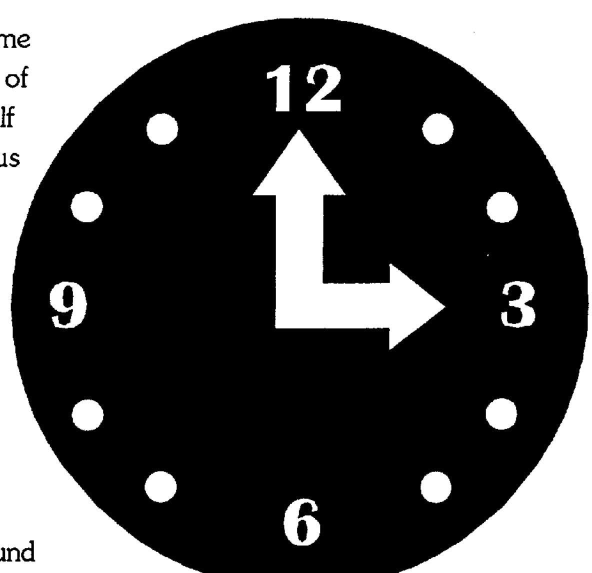
Since there are twelve numbers around the clock, it's easy to see that each number marks 1/12 of the clock's face. This gives her some useful fraction combinations. The number 4 (for the fraction 1/3) comes right after the number 3 (for 1/4), which means that $\overline{3} = \overline{4}\ \overline{12}$.

And of course, the number 9 (for 3/4) comes right after 8 (which is 2/3). This gives the equation Alex figured out earlier for Dr. Jones: 3/4 = $\overline{\overline{3}}\ \overline{12}$.

Fifths and tenths can work on a fraction clock, too. A tenth of 60 is six minutes. So to find 3/10 on the clock, Alex counts by sixes: 6, 12, 18 minutes.

Each fifth is 2/10, or 12 minutes on the clock. This explains the second equation Alex used for 3/5: Counting by twelves, 3/5 is 12, 24, 36 minutes, or half an hour plus six minutes. That gives 3/5 = $\overline{2}\ \overline{10}$.

She hasn't been able to figure out an easy way to picture sevenths or ninths. Even so, Alex's clock trick is a useful tool for making abstract fraction work easier. Don't you agree?



Jean Baptiste Joseph Fourier (1768-1830) was a mathematical physicist who wanted to find equations to describe nature. He studied under Monge. Then he went with Napoleon to Egypt, where he worked as a diplomat. Fourier wanted to analyze heat energy, but the problem was too complex. So he invented the Fourier series, which turns a complicated equation into the sum of simpler functions.