# Let's Play MATH

## How Homeschooling Families Can Learn Math Together, and Enjoy It!

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Tabletop Academy Press

Tabletop Academy Press, Blue Mound, IL, USA.
Parts of this book were originally published in 1998, 2000, under the title *Aha! How to Teach Math So Kids Get It*.

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## Acknowledgements

No man is an island, and authors perhaps less than most. I cannot possibly express the debt I owe to the family, friends, and online acquaintances whose help and support have made this book possible. Special thanks to my husband David, whose patience stretches far beyond what I deserve, and our children, who taught me so much over the many years of homeschooling.

Thank you to Marilyn Kok, who pushed me to write way back in the beginning, to Julie Pangrac, who talked me into doing workshops for homeschool groups, and to Sue VanHattum, whose work on her own book, *Playing With Math: Stories from Math Circles, Homeschoolers, and Passionate Teachers*, convinced me to bring my books back into print. To my many thousands of book and blog readers, your comments and encouragement have kept me going. To my fellow math bloggers, I've learned so much from you all!

Fervent unending gratitude goes to my beta readers, especially to Maria Muscarella and Becky Johnston, and to my editor Jim Mcfarlin—whatever mistakes remain are due to my pigheadedness in overruling their advice.

# Contents

It is in fact nothing short of a miracle that the modern methods of instruction have not entirely strangled the holy curiosity of inquiry; for this delicate little plant, aside from stimulation, stands mainly in need of freedom.

-Albert Einstein

# Introduction

Can you separate educational fact from fiction? Which of the following statements are true, and which are math myths?

- Math is logical and rigid, never creative or artistic.
- In mathematics, answers are either right or wrong. The right answer is never a matter of opinion.
- Math is timeless and objective. It's the same for everyone.
- Math is in the genes. Some people have a "math mind," but most of us don't.
- Children should never count on their fingers.
- Learning mathematics is like climbing a ladder. You have to master the basics before you can reach the higher rungs.
- Looking at someone else's answer is cheating.
- Students should show all the steps of their work. Shortcuts will lead to mistakes.

- Children need to memorize the times table. They should drill the math facts until they can answer flashcard-fast.
- You need a textbook to help you learn math.

Actually, the quiz is rigged. I have seen most of the above statements in various homeschooling books, websites, or forums, yet not one of these statements is indisputably true.

Mathematics is much more than a set of rules, and it can be very artistic. Math is not the same for everyone, because what you see depends tremendously upon your point of view. For example, can you draw a pair of parallel lines? In some versions of geometry, parallel lines do not exist. Does 2 + 2 = 4? Not always: in mod 4 arithmetic 2 + 2 = 0, while in base 3 it equals 11.

The myths related to teaching may have a semblance of truth, but their cumulative effect is to limit our children's understanding and ability to appreciate math.

Many later math topics do build on earlier ones, but learning math is more like a meandering stream than like a ladder with one rung above another. Preschool children are capable of exploring topics like fractals and infinity, while early elementary students can begin learning algebra. To play with advanced ideas is fun, and such adventures give our students a broader perspective that reinforces and builds up their knowledge of the more standard arithmetic topics.

Or consider the idea that looking at someone else's answer is cheating. While that is undoubtedly true during a test, such pressure should be rare in a homeschool. Wise parents know that, when their students are stumped on a math exercise, one of the best ways to learn is to look up the answer and work backwards.

When I was a student, teachers universally required us to "Show Your Work!", and I've seen many forum threads asking how parents can convince students to write out the steps of their answers. Yet one of the great freedoms we have as homeschoolers is that we can take the time to sit and talk with our children. I can ask my daughter, "How did you figure it out?" In the course of

conversation, she will demonstrate very clearly how much she knows. I often find myself learning something from the discussion, too, because the way she thinks a problem through is usually different from what I would have done. I'd hate to trade that opportunity for a notebook page full of written-out steps.

Even the statement about memorizing the times table is a myth, perhaps the most widely-believed myth of them all. Day after day, homeschool forums abound with parents desperate to know, "How can we get our children to learn the math facts?"

It is the wrong question. While math facts are definitely useful, other things are more important.

When parents stress memorization too early, we short-circuit the child's learning process. Once our children "know" an answer, they no longer bother to think about it. It's better for children to spend plenty of time in the "thinking about it" stage, because that will allow them to build a logical foundation for understanding not only the math facts but many future topics as well.

Nor is a textbook necessary, at least during the early-elementary years. Children have an intuitive feel for many math concepts. Most young children are fascinated with numbers, especially big numbers like hundreds or thousands. They enjoy drawing circles and triangles, and they delight in scooping up volumes in the sandbox or bathtub. They can count out forks and knives for the table, matching sets of silverware with the resident set of people. They know how to split up the last bit of birthday cake and make sure they get their fair share, even if they have to cut halves or thirds.

*Let's Play Math* will help you see how to build on your children's natural attraction to mathematical ideas and help them develop problem-solving skills. You will discover how students of all ages can enjoy learning math, no matter what math program your family uses—or whether you use a formal curriculum at all.

[You will find references for all the quotations plus assorted other tips and useful links in the appendix Notes and References.]

I used to think that math was some kind of inaccessible, abstract magic trick, a sort of in-joke that excluded us common folk, but now I realize that math is completely not that at all. The reality of math as most of us know it is like that story where three men are standing in a dark room touching different parts of an elephant. None of them has the full picture because they're only perceiving individual elements of the whole animal.

The reality, I'm discovering, is that math is just like that elephant: a large, expansive, three-dimensional, intelligent, sensitive, expressive creature.

The problem is that most of us have been standing around in that dark room since about kindergarten, grasping its tail, thinking "this is what math is and, personally, I don't think it's for me." We've been blind to the larger, incredibly beautiful picture that would emerge if only we would turn on the lights and open our eyes.

-Malke Rosenfeld

# 1

If a child is to keep alive his inborn sense of wonder, he needs the companionship of at least one adult who can share it, rediscovering with him the joy, excitement, and mystery of the world we live in.

-Rachel Carson

## The "Aha!" Factor

I sat on the bed, surrounded by stacks of notes, bills, and other papers. Our six-year-old bounced onto the other side.

"Mom, can we do some math?" she asked.

We delay academics, so I would not normally have done schoolwork with her at all. I suppose she was jealous of what she saw as her older siblings' Mommy-time.

I started to say I was busy, but stopped myself in mid-grumble. *You do that too often,* I scolded myself, and forced a smile.

"OK," I said. "Let's see what we can find."

I put the most important papers safely to the side. We counted the things that remained, then took some away and counted again.

Then I leaned over and whispered in her ear. "Guess what? We made cookies, just for you and me. And we're not going to share them with the big kids."

Her eyes grew wide. "Really?"

I told her we had six (imaginary) cookies. I piled up two notebooks, a used envelope, a sheet of scratch paper, a computer printout, and my pen. She

divided the "cookies" between us, and we giggled as we pretended to eat. Then she picked out seven new cookies for me to divide. I made exaggerated motions of cutting the electric bill in half. We counted things one by one and in pairs, paying attention to which numbers came out even and which numbers made us cut up the last cookie.

After 10 minutes she went away happy, and I returned to my work. Subtraction, division, even/odd, fractions … In that short time, we had touched on more math than we might have found in a week's worth of workbook pages.

For young children, mathematical concepts are part of life's daily adventure. Their minds grapple with understanding abstract ideas like *threeness*: the intangible yet real link between three blocks and three fingers and three raisins on a plate. But after a few sessions of "3 + 1 = 4, 3 + 2 = 5, 3 + 3 …" they begin to whine. Older children recoil from long division. By the time they reach high school, students are faced with torture like: "The product of an integer and the next greater integer is 20 less than the square of the greater integer." Math becomes a tedious chore to put off as long as possible or finish with slapdash speed.

Mathematics should be a game of discovery. It should give children the same *Eureka!* thrill that sent Archimedes running through town in his birthday suit. I call this *the "Aha!" factor*, the delight of solving a challenging puzzle. This "Aha!" factor is what I aim for when I bring home a brainteaser book from the library or a new game from the store. And it is what I find emphatically missing from most math textbooks.

## The Problem with School Math

Why, as they grow up, do so many children learn to hate math? And why do so many homeschool parents feel inadequate to teach math?

American mathematician Hassler Whitney once said that it is "no wonder you hate math. You never had a chance to see or do real math, which is easy and fun."

Easy?! Yes, that is what he said. Of course, some parts of math can be hard to understand and some math problems are fiendishly difficult. But compared to school math, which requires us to memorize and recall artificial rules for the manipulation of abstract quantities, real mathematics is closely related to common sense—which makes it relatively easy.

As British mathematician and educator W. W. Sawyer explained, "A widespread fallacy about teaching is the idea that remembering is easy and understanding difficult. John is a bright boy, we will teach him what the subject really means; Henry is dull, he will just have to learn things by heart. Now exactly the opposite is true: to remember things which you do not understand is extremely difficult."

Real mathematics is intriguing and full of wonder, an exploration of patterns and mysterious connections. It rewards us with the "Aha!" feeling. These characteristics make it easy to stick with real math, even when a particular concept or problem presents a difficult challenge.

Real math is the surprising fact that the odd numbers add up to perfect squares (1, 1 + 3, 1 + 3 + 5, etc.), and the satisfaction of seeing why it must be so. Try it for yourself:

- Gather a bunch of small blocks—or any small items that will not roll away when you set them out in rows—and place one of them in front of you on the table. Imagine drawing a frame around it: one block makes a (very small) square.
- Now, put out three more blocks. How will you add them to the one in order to make a new, bigger square?
- Set out five additional blocks. Without moving the original four, how can you place these five to form the next square?
- Then how many will you have to add to make the square after that?
- Can you see that the "next odd number" pattern will continue as long as there are blocks to add, and that it will keep going forever in imagination?

Textbook math, on the other hand, is page after page of arithmetic. Textbook math is long division by hand, followed by a rousing chorus of the Fraction Song: "Ours is not to reason why, just invert and multiply."

School ruins mathematics for most people, distorting a discipline that is half art and half sport by turning it into successive rounds of memorization and testing. Can you imagine a piano teacher who insisted her students spend six years on scales and exercises of gradually increasing difficulty before she would let them attempt a piece of actual music? Or a football coach who made his team run laps and do sit-ups every day, but only let them play two or three games a year, and scrimmage games at that. How many people would become bored with music or learn to hate football under such instruction?

As every coach knows, skill grows through practice, practice, practice. But practice has no meaning unless the team has a real game to play. And the best type of practice takes advantage of the benefits of cross training by emphasizing variety rather than repetitive drills. Mathematical cross training will include games, puzzles, stories, patterns, and the task of thinking things through.

Of course, our students do need to learn how to perform routine calculations, just as piano players must practice scales and football players lift weights. More important, however, our students need to learn why those operations work.

And they should never, ever be led to think that calculations are the essence of mathematics.

"A teacher of mathematics has a great opportunity," wrote Hungarian math professor George Pólya. "If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students, he may give them a taste for, and some means of, independent thinking."

## Playing with Numbers

Writing for *Family Life* magazine, mathematician and music critic Edward Rothstein described a game he invented for his daughter:

"What number am I? If you add me to myself, you get four."

I gave that question to my six-year-old daughter during a family car trip. Then her sister, age nine, wanted in the game. I tried a question with slightly bigger numbers, but she rolled her eyes. "That's too easy, Mom."

So I asked her:

"What number am I? If you take away one fourth of me and then add two, you get 17."

[You can find Answers to Sample Problems in the appendixes. This is problem 1. It has two possible answers, depending on how you understand the words in the question. Of course, my daughter did not see it the same way I did. Let this be a warning: *if your child's answer is not the same as yours, don't automatically assume she is wrong!* Ask her to explain how she figured it out, and then listen carefully. Children usually have a logical reason for their answers. Language is a complicated thing, and even a math problem may be open to different interpretations.]

The older they get, the harder a parent has to work. For my twelve-year-old son, I asked:

"What number am I? If you multiply me by myself and add one, you get half as many as the number of pennies in a dollar." [2]

That kept him busy for a few minutes. After he figured it out, he came back with:

"What number am I? If you divide me by two and take away four, then add five, then multiply by three and divide by two and add seven, you get me again."

"What?"

He repeated the question.

"This is actually a number?" I asked. "You figured out an answer to this?"

He nodded, with the smug grin of a preteen who knows he has Mom skewered.

I pulled out a notebook and pen. He repeated his series of calculations, and this time I wrote it down. I figured the answer had to be zero or one, those magic numbers that make multiplying easy, but neither worked.

I tried 100. No luck.

I heard a chuckle from the back seat.

"Wait," I said. "Give me a chance."

My husband was driving, but he glanced over at the notebook. "You know," he offered, "you could set that up as an equation."

No way. The boy had not needed algebra to figure it out, so neither did I. I tried 10, then 50, then 20. OK, that narrowed it down. Now I knew it was between 20 and 50, but I had run out of easy numbers.

I nibbled on the end of my pen.

My son hummed to himself.

"I've got it!" I spun around as far as the seat belt allowed. "The answer is—." [3]

"Nope."

"WHAT?!"

I looked at my scratch paper and worked the numbers again, coming up with the same answer. I read the steps of my calculation out loud.

He agreed that my number would work, but it was not the one he had in mind. I would have to guess again.

Hubby protested that there couldn't be another answer. If the equation doesn't have an $x^2$ or something similar, there cannot be more than one solution.

The kid stood his ground, smirking.

I conceded. "What's your number?"

"Infinity! It doesn't matter what you multiply or take away, it's always infinity."

Aha! He was right. Well, sort of right: you can't really calculate with infinity that way, but it's good enough for middle school. Even better, he had a chance to stump the teacher. You won't find that in a sixth-grade textbook.

## What Is Our Goal?

Many parents begin their journey into homeschooling by writing out (or at least talking through) their philosophy of education. Is our mission more like filling the empty bucket of a child's mind, or like lighting a fire that will grow and spread on its own? Or is our role not to "teach" at all, but rather to walk alongside and assist our children as they explore and learn about the world? How we define our goals will make an enormous difference in the way we approach the day-to-day adventure of homeschooling.

In the same way, before we can figure out how to help our children learn math, we need to think about our goals. What does it take to understand mathematics, and is it truly necessary for our children? After all, don't computers and calculators crunch most of the numbers in the world? Of course, some children must grow up to program those computers, but what if my kids have other plans? People say that worksheets help students learn the discipline of sticking with an unpleasant task, but surely chores develop that sort of perseverance just as well.

Am I teaching math simply because the state requires it? The state requires our children to study math so they will be functionally literate. That may not sound like a lofty goal, but think about what "functionally literate in math" means:

- Filling out an IRS Form 1040 with its Schedule A, Schedule B, Schedule SE, and all the rest.
- Reading a mortgage, and understanding how a fixed- or variable-rate loan will affect family finances.
- Following newspaper articles about the governor's budget proposal, or discerning the relevance of political polls.

- Knowing that a 40% chance of rain on Saturday and a 60% chance of rain on Sunday doesn't mean there is a 100% chance of rain this weekend.

Mathematical literacy is a worthy challenge. But most of us want more than this for our children. We want them to be *educated*. An educated person studies more than merely what is useful. He or she loves to learn, studies for the sake of gaining knowledge, and grows in wisdom.

As math educator Paul Lockhart writes, "Just because a subject happens to have some mundane practical use does not mean that we have to make that use the focus of our teaching and learning. It may be true that you have to be able to read in order to fill out forms at the DMV, but that's not why we teach children to read."

Very few people read Shakespeare because his plays are useful. But an educated person will read Shakespeare because his stories are interesting and his dialogue insightful. Likewise, much of math does not seem to be immediately useful, but it can be extremely interesting.

Consider fractals, those intricate, computer-generated patterns of regular irregularity that capture something of the complex beauty in nature. The men who first discovered fractals in the late 1800's called them "monster curves" and could not imagine any use for them. Yet today, these monsters are used to compress images and other files and have become a staple tool in the movie maker's special-effects kit.

Children who are educated in math will undoubtedly gain practical skills. But what is more important, those who enjoy learning for its own sake will find plenty to fascinate them.

## Math the Mathematician's Way

Too many people think that the essence of mathematics is "2 + 2 = 4" and knowing the times tables by heart, or that being good at math means being able to pass a machine scored, multiple choice standardized achievement test. No,

no, and certainly not! Listen to how real mathematicians, both professionals and amateurs who actually enjoy working with math, describe their subject:

> Many who have never had the occasion to discover more about mathematics consider it a dry and arid science. In reality, however, it is a science that demands the greatest imagination.
>
> -Sofia Kovalevskaya

> Real mathematics is not just formulaic tutoring. My hope is that children learn to think about mathematics as a kind of mental play.
>
> -Edward Rothstein

> Puzzles are made of the things that the mathematician, no less than the child, plays with, and dreams and wonders about, for they are made of the things and circumstances of the world he lives in.
>
> -Edward Kasner

> There is no ulterior practical purpose here. I'm just playing. That's what math is: wondering, playing, amusing yourself with your imagination.
>
> -Paul Lockhart

> If mathematics education communicated this playful aspect of the subject, I don't think innumeracy would be as widespread as it is.
>
> -John Allen Paulos

Basic mathematics is mental play. This is the essence of creative problem solving. This is what we need to teach our children, more important than fractions or decimals or even the times tables. Math is a game, playing with ideas.

W. W. Sawyer wrote a book called *Mathematician's Delight*. He described mathematical thinking this way: "Everyone knows that it is easy to do a puzzle if someone has told you the answer. That is simply a test of memory. You can

claim to be a mathematician only if you can solve puzzles that you have never studied before. That is the test of reasoning."

It is also the test of life. Every day we face problems we have not studied before. Math taught the mathematician's way prepares us to approach problems creatively, to see our mistakes as stepping stones to learning, and to recognize that there may be more than one right answer, as my children and I discovered when we played "What Number Am I?"

Math taught the mathematician's way gives our students practice struggling with challenging problems and lets them enjoy that "Aha!" thrill when they find a solution. Math taught the mathematician's way prepares them for careers or college. It gives them tools they can use throughout their lives. It gives them confidence by letting them succeed at something difficult. When children solve a puzzle that stumped Mom or Dad, they know they can handle anything.

Math taught the mathematician's way is a social adventure of exploring and sharing new ideas. Math taught the mathematician's way is fun, and it can even be beautiful.

Math taught the textbook way, however, prepares children to parrot answers.

Math taught the textbook way drives people to agree with Huckleberry Finn: "I had been to school … and could spell, and read, and write just a little, and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could ever get any farther than that if I was to live forever. I don't take no stock in mathematics, anyway."

Children who play around with math taught the mathematician's way learn to solve problems, though they may not learn every concept at the exact time demanded by their state's math standards. Homeschool parents have the freedom to de-emphasize worksheets and timed drills, to avoid the anxiety-producing pressure of tests, and to encourage our children to focus on reasoning skills and thinking a situation through. We have the freedom to skip quizzing our child on the addition facts or times tables while we explore wonder-inducing ideas like Fibonacci numbers (see *Fibonacci's Rabbit Problem* in

Chapter 6) or exponential growth (see the *Penny Birthday Challenge* in Chapter 3).

Some have criticized this approach as "fuzzy" math, akin to the oft-maligned "whole language" approach to reading. They claim that children who spend less time practicing computation skills will not perform as well on standardized tests. Even more important, critics charge that such "dumbing down" of the curriculum will make our students fall farther behind their international peers. They mock non-traditional math programs, implying that to teach math without flashcard drills is the same as saying, "2 + 2 = 5 if that makes you feel happy."

Where is the truth? The only children who could answer that 2 + 2 = 5 are the ones who have been *discouraged* from thinking a problem through. Kids know better than that! Can you imagine your child letting someone get away with math like that in a basketball game?

Of course our children need to know how to add and subtract, multiply and divide. But Fibonacci numbers give students an adventure in real-world addition that goes far beyond the so-called math facts. And exponential growth demonstrates real-world multiplication in a way that even many adults do not understand.

Flashcard drills and practice pages of arithmetic problems are the least effective ways to teach math. Rote learning is the laziest, most deadening way to approach any subject. Think of memorizing dates in history class, and compare that to reading historic speeches and well-written biographies. On occasion, such memory work may be necessary, but it should never be the bulk of a student's experience in any subject.

Instead, we need to introduce our students to the thrill of tackling tough, challenging puzzles. We need to give children a taste of the joy that comes from figuring things out, the "Aha!" factor. Learning to think a problem through and get the solution can be hard work—and that is exactly what makes it fun.

# How Can I Teach Math If I Don't Understand It?

Our childhood struggles with schoolwork gave most of us a warped view of mathematics. We learned to manipulate numbers and symbols according to what seemed like arbitrary rules. We may have understood a bit here and a bit there, but we never saw how the framework fit together. We stumbled from one class to the next, packing more and more information into our strained memory, until the whole structure threatened to collapse. Finally we crashed in a blaze of confusion, some of us in high school algebra, others in college calculus.

Yet even parents who suffer from math anxiety can learn to enjoy math with their children. All it takes is a bit of self-discipline and the willingness to try. If you are afraid of math, watch your tongue: don't let a discouraging word pass your lips. Pretend like math is the most exciting topic in the world. Encourage your children to notice the math all around them. Investigate, experiment, estimate, explore, measure—and talk about it all. Curl up together on the couch to read a math book, or tell math stories at bedtime. Look for opportunities to discuss numbers, patterns, shapes, and symmetry, not as a "homework" exercise but just because they are interesting.

The secret to teaching homeschool math may surprise you: we parents need to learn how to think like a mathematician. Mathematicians avoid busywork as if it were an infectious disease. Mathematicians always ask questions, and most of all, they love to play. If parents cultivate these characteristics, we will help our children to recognize and learn true mathematics.

## Mathematicians Are "Lazy"

Mathematicians are not indolent, but economical with their time and energy. They have too many interesting topics to study and not enough time to learn about them all, so they cannot afford to waste their time on mindless busywork.

Mathematicians are always looking for a simpler way to do things. Skip counting is faster than addition; multiplication is even easier. Algebraic functions are shorthand ways to say things like, "Take any number you like and multiply by five, then subtract four, and divide it all by seven, and add one." Calculus lets engineers solve problems that would be impossible without it.

Some problems have more potential solutions than anyone could list, so mathematicians invented special ways to think about counting. *Permutations* let us count the possibilities when the order matters: "How many different ways might we award the 1st, 2nd, and 3rd place prizes in a race?" *Combinations* count the ways to do something when order doesn't matter: "How many different three-person committees might we choose to plan the party for our homeschool co-op?" Both types of puzzle can be fun for middle school or older students.

When we look at education through the lens of mathematical laziness, we will be skeptical about the value of repetitive practice problems. If our children keep forgetting how to do certain calculations, perhaps they are not yet developmentally ready to learn them. One of the biggest problems in math education is how often teachers train students to do "number tricks" the same way we would train a dog to fetch or roll over, by having them repeat the procedure until they can do it without thinking. Children will master the math better if we wait until they have developed a foundation that will help them understand why it works.

In the meantime, there are ever so many interesting ideas we can explore together. Parents who think like mathematicians will always make time for the fun stuff.

## Mathematicians Ask Questions

Wise mathematicians are never satisfied with merely finding the answer to a problem. If they decide to put effort into solving any math puzzle, then they are determined to milk every drop of knowledge they can get from that problem. When mathematicians find an answer, they always go back and think about the

problem again. Is there another way to look at it? Can we make our solution simpler or more elegant? Does this problem relate to any other mathematical idea? Can we expand our solution and find a general principle?

As math teacher Herb Gross says, "What's really neat about mathematics is that even when there's only one right answer, there's never only one right way to do the problem."

School textbooks only ask questions for which they know the answer. When parents learn to think like mathematicians, we will ask a different type of question. Try asking your children (and encouraging them to ask) questions to which you don't know the answer, questions like:

- What do you think?
- What do you see?
- How did you figure that out?
- Is there another way to look at it?
- Will this always be true?
- Can we predict what will happen next?
- Is there a pattern?
- Will the pattern continue, or will it run out?
- How can we be sure?
- What would happen if ___?
- Why?

As your children try to put their thoughts into words, keep in mind this truth:

> Most remarks made by children consist of correct ideas very badly expressed. A good teacher will be very wary of saying "No, that's wrong." Rather, he will try to discover the correct idea behind the inadequate expression. This is one of

> the most important principles in the whole of the art of teaching.
>
> -W. W. Sawyer

Don't worry if you can't find the answers to all of the questions you or your children ask. Some mathematical questions have taken centuries to answer and led to entirely new branches of study. In the quest of learning math, wondering can be its own reward.

## Mathematicians Love to Play

Mathematicians play with ideas. They toy with puzzles. They tinker with the connections between shapes and numbers, patterns and logic, growth and change. To a mathematician, the fun of the game is in experimenting, in trying new things and discovering what will happen. Many modern strategy games were invented primarily for the fun puzzle of analyzing who would win.

For example, consider the simplest form of the two-player strategy game *Nim*. Start with a pile of pennies, and take turns removing either one or two pennies from the pile until one player is forced to take the last coin, thus losing the game. Play the game with your children several times, and then encourage them to think of some way to change the rules. Ask questions: How does the game change? Is it easier or harder to win? Does the player who goes first have an advantage?

Parents who think like mathematicians will join their children in playing with numbers, shapes, and patterns. We might pick a number and double it, and then double that, and keep doubling to see how high we can go. Or we may build a perfect square pyramid out of sugar cubes (with 1, 4, 9, 16, 25, 36, and 49 cubes in the layers) as the centerpiece for our next tea party. Or we could think about new ideas: a point has no dimension, but if we imagine pulling hard to stretch it out in one direction, it would become a one-dimensional line. A line pulled out and stretched into two dimensions becomes a square, and a square stretched into three dimensions becomes a cube. So what will happen if we could stretch a cube into four dimensions?

Don't be like the high school algebra teacher who caught a student playing tic-tac-toe and snatched away his paper, saying, "When you're in my classroom I expect you to work on mathematics!" When the boy's friend, popular math writer Martin Gardner, heard the story, he responded that tic-tac-toe was an excellent introduction to symmetry, probability, set theory, *n*-dimensional geometry, and other topics. "With a little guidance," Gardner said, "it might have been much more rewarding than what his teacher was teaching."

Learning to think like a mathematician is a lifetime adventure. I won't have room in this book to mention perfect numbers, topology, Penrose tiles, cryptograms, or any of a zillion other topics just waiting to be explored. When you begin to look at the world with a mathematician's eye, you embark on a journey more varied than the voyage of Ulysses, more exotic than Marco Polo's travels, and more adventuresome than a trip to the moon.

# Ready for More?

If you enjoyed this excerpt and would like to discover more great ways to turn math into a learning adventure for the whole family, look for *Let's Play Math: How Homeschooling Families Can Learn Math Together, and Enjoy It!* at your favorite online bookstore:

- Find the book at these retailers: Amazon.com; iTunes bookstore; Barnes & Noble; Kobo; or Smashwords
- Or buy direct from the author: Let's Play Math at Ganxy

Your children will build a stronger foundation of understanding when you teach math as a game, playing with ideas.

## Reviews of *Let's Play Math*

Combined with all the linked resources, this book is going to transform how I teach my kids maths. No more dabbling in "real maths" but then running back to the workbooks when anxiety strikes — with this approach I can teach my kids to think like mathematicians without worrying about leaving gaps.

…

My favourite section of the book is "One Week of Real Mathematics", which contains examples of what one week's worth of math playtime might look like. I love having this starting point to show me what a balanced "maths diet" might look like.

…

I knew the well-travelled road (maths curricula) wasn't for us, but I lacked confidence in my ability to guide my children through uncharted territory. Let's Play Math is the map and the guidebook I've been looking for. With it in my hand I can't wait to take my children by the hand and head off to explore the wonderful world of maths.

— Lula B

Navigating By Joy

This book is a creative and inspiring challenge to the way maths is usually taught in school. It is packed full of ideas and resources to help children (and definitely adults too!) develop their learning about the art of numbers. In a culture where maths anxiety is now a diagnosable problem, this book shows the way to maths joy.

— J. Mcandrews

This book will change the way you look at math forever … *Let's Play Math* challenges homeschooling parents to teach their kids to 'think like mathematicians' and use their problem solving skills to really understand concepts rather than just memorize processes … This is definitely a must-read ebook for all homeschoolers!

— Learners in Bloom

I love math, but had forgotten why I developed a love for math in the first place. This book … shows us how we can ignite this fire in our own children.

— Carrie

I loved how this book reframes our concepts of Math … helps parents understand what Math really is about, what fun there is in it … games and kinds of behavior they can adopt to develop a Math sense in their children.

— Rafael Falcón

It is well researched, well annotated, and includes loads of activities that you can try with kids K-12 at home. While reading the book, I found myself remembering a lot of things I had forgotten from my teacher-training in Constructivist math.
...
There were so many parts of this book that I highlighted that I really gave my Kindle a workout! But what was especially useful to me at this moment, were the talking points for helping kids solve problems on their own. I'm going to print up all of the talking points and post them in our kitchen so that my husband and I will have a list of questions to prompt our son's thinking.

— Jennifer Bardsley