Cooperative Problem Solving

The Centauri Challenge

For the past sixty years, astronomers of the lunar colony have been communicating with an intelligent life-form in the Centauri star-system. Because of the vast distances, communication is very slow. It takes twenty years for a message to be sent and a response received. The most recent communication included *The Centauri Challenge*. *The Centauri Challenge* was sent from logic students of the Centauri star system to students at the lunar colony. The challenge is shown below.

The Centauri Challenge

Centauri is a formal system of strings of the letters *P*, *Q*, *R*, *S* with four rules governing their behavior. Using some combination of the four rules, it is possible to change one string of letters into a different string. The four rules are:

Rule 1: Any two adjacent letters in a string can change places with each other. (*PQ* >> *QP*)

Rule 2: If a string ends in the same two letters, then you may substitute a *Q* in place of those two letters. (*RSS* >> *RQ*)

Rule 3: If a string begins in the same two letters, then you may add an *S* in front of these two letters. (*PPR* >> *SPPR*)

Rule 4: If a string of letters starts and finishes with the same letter, then you may substitute an *R* in place of all the letters between the first and last letters. (*PQRSP* >> *PRP*)

A theorem in this system is a string followed by a >> followed by a string. For example, *PQQRSS* >> *QRQ* says, if given string *PQQRSS*, then by using the rules you can arrive at *QRQ*. An example of the proof of this theorem is shown below.

Example

Show: *PQQRSS* >> *QRQ*

Proof:

PQQRSS	Given
PQQRQ	By Rule 2
QPQRQ	By Rule 1
QRQ	By Rule 4

Therefore, *PQQRSS* >> *QRQ*.

Challenge 1:

Use the rules of Centauri to prove the theorems below.

1. *PQPRQ* >> *RQ*
2. *PQRSSQR* >> *RQ*
3. *PSSRS* >> *RQ*
4. *PSRQQRSQPSSS* >> *RQ*
5. *PQQQQP* >> *RQ*
6. *QQQQQQ* >> *SRQ*

Challenge 2:

Use the rules of Centauri to answer the questions below.

1. Can you find a string of five or more letters that *cannot* be reduced to *RQ*? If yes, produce it. If not, prove that you cannot.
2. One of the rules of Centauri can be removed without losing any of the first five theorems proved in the first challenge. Which one of the rules can be removed? Why must this rule be used in the sixth theorem? Create another theorem that must use this rule.